Phase Space Description of the Leading Order Quark and Gluon Production from a Space-Time Dependent Chromofield

Dennis D. Dietrich, Gouranga. C. Nayak, and Walter Greiner

Institut für Theoretische Physik, J. W. Goethe-Universität, 60054 Frankfurt am Main, Germany

Abstract

We derive source terms for the production of quarks and gluons from the QCD vacuum in the presence of a space-time dependent external chromofield A_{cl} to the order of $S^{(1)}$. We found that the source terms for the parton production processes $A_{cl} \to q\bar{q}$ and $A_{cl}, A_{cl}A_{cl} \to gg$ also include the annihilation processes $q\bar{q} \to A_{cl}$ and $gg \to A_{cl}, A_{cl}A_{cl}$. The source terms we derive are applicable for the description of the production of partons with momentum p larger rhan gA which itself must be larger than Λ_{QCD}. We observe that these source terms for the production of partons from a space-time dependent chromofield can be used to study the production and equilibration of the quark-gluon plasma during the very early stages of an ultrarelativistic heavy-ion collision.

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I. INTRODUCTION

Recently, a lot of effort is made to study the production and equilibration of the quark-gluon plasma in ultrarelativistic heavy-ion collisions at RHIC and LHC [1]. Such a state of matter is predicted by lattice QCD calculations at high temperatures and high densities [2]. The major problems one encounters in such a study is how quarks and gluons are formed in these experiments and how their phase-space distribution evolves in space-time with collisions among the partons taken into account. At ultrarelativistic energies, the two nuclei are highly Lorentz contracted. When they pass through each other, a chromoelectric field is formed due to the exchange of soft gluons [3–7]. This is a natural extension of the color flux-tube or the string model which are widely applied to high energy pp, e^+e^- and pA collisions [8,9]. This chromoelectric field polarizes the QCD vacuum which leads to the production of quark/antiquark pairs and gluons via a Schwinger-like mechanism. These quarks and gluons collide with each other to form a thermalized quark-gluon plasma. The space-time evolution of the formed partons can be studied by solving the relativistic non-abelian transport equations for quarks and gluons [5,10]. As the chromofield exchanges color with quarks and gluons, color is a dynamic quantity. The time evolution of the classical color charge follows Wong's equations [11]

$$\frac{dQ^a}{d\tau} = f^{abc} u_\mu Q^b A^{c\mu} \tag{1}$$

and

$$\frac{dp^\mu}{d\tau} = Q^a F^{a\mu\nu} u_\nu. \tag{2}$$

Here, the first equation describes the precession of the color charge in the presence of a classical background chromofield. The second equation is the non-abelian version of the Lorentz-force equation. Taking Wong's equations into account, one finds the relativistic non-abelian transport equation [5,10] which for quarks reads for example as:

$$\left[p_\mu \partial^\mu + g Q^a F^a_{\mu\nu} p^\nu \partial^\mu_p + g f^{abc} Q^a A^b_\mu p^\mu \partial^c_Q\right] f(x,p,Q) = C + S. \tag{3}$$

Note that there are different transport equations for quarks, antiquarks, and gluons. In the equations mentioned above, $f(x,p,Q)$ is the single-particle distribution function of quarks, antiquarks, and gluons respectively in the 14-dimensional extended phase space of coordinate, momentum and color in the $SU(3)$ gauge group. The first term on the LHS corresponds to common convective flow, the second term is the non-abelian generalization of the Lorentz force term and the third term represents the precession of the color charge in the presence of the background field as described by Wong's equations. On the RHS, C denotes the collision term which describes the collisions among the partons and S represents the source term for the production of the respective particle species. The source term S contains the detailed information about the creation of partons from the chromofield. It is defined as the probability for the production of a parton per unit time per unit volume of the phase space. For this reason, the development of the QGP is fundamentally governed by this source term S.

The formation and equilibration of the quark-gluon plasma within the color flux-tube model is studied by several authors [4,5]. However, in all these studies, source terms for parton production from a constant chromoelectric field were employed, because source terms for parton production from a space-time dependent chromofield were not studied in literature before. In fact, the chromofield acquires a strong space-time dependence as soon as it starts producing partons. Also the acceleration, collision and precession terms present in the transport equation make the field space-time dependent. This implies that the source term for particle production from a constant field is not applicable under these circumstances.

We mention here that particle production from the classical field via vacuum polarization is studied in two different cases. In the presence of a constant uniform background field the particle production is computed through the Schwinger mechanism [12] which is an exact one-loop non-perturbative result. This result can also be understood as semi-classical tunneling across the mass gap [13]. However, for a space-time dependent field, particles can be produced directly by a perturbative mechanism [12,14]. It is observed in numerical studies [5] that the chromofield acquires a strong space-time dependence as soon as it produces partons. Due to these reasons, formulas based on constant fields are not applicable to such problems and source terms for parton production from a space-time dependent chromofield are necessary. In this paper, we will derive the source terms for the creation of $q\bar{q}$ and gg pairs from a space-time dependent chromofield in order of $S^{(1)}$.

The production of $q\bar{q}$ pairs from a space-time dependent non-abelian field is similar to the creation of e^+e^- pairs from an abelian field. Apart from the color factors, the interaction of a quantized Dirac field with the classical potential is the same in both cases. So, the results obtained for the probability of creation of e^+e^- pairs from a Maxwell field can be transfered to the creation of $q\bar{q}$ pairs from a Yang-Mills field. Contrary to that, the production of gluons from a classical external Yang-Mills field has no analogue in the abelian case. There exist direct interactions between the quantized non-abelian field and the classical non-abelian potential which then lead to the production of gluons from the QCD vacuum. In this paper, we will derive the source term for the production of gluons from a space-time dependent chromofield through the application of the background field method of QCD which was developed by DeWitt [15] and later on extended by 't Hooft [16].

As already mentioned before, in the color flux-tube model the creation of the chromofield is via soft gluon exchange when two nuclei cross each other [3–7]. It is the extension of the string model widely applied to high-energy e^+e^--, pp- and pA- collisions [8,9]. However, this is not the only model where a chromofield is created in high energy heavy-ion collisions. The McLerran-Venugopalan model also predicts the existence of a chromofield at low x and low transverse momentum [17,18]. Also recent work by Makhlin and Surdotovich [19] and many others, *e.g.* [20,21] seems to indicate that the highly gluon dominated environment in heavy-ion collisions will be ruled by scales resulting from the presence of background fields. In any case, once there is a chromofield it will polarize the QCD vacuum and will produce quarks and gluons which will then form a thermalized quark-gluon plasma. In the McLerran-Venugopalan model the parton production is described as classical radiation whereas we describe parton production via vacuum polarization. It might be interesting to incorporate gluon creation both by classical radiation and vacuum-polarization in the transport equation to study the evolution of the quark-gluon plasma.

The paper is organized as follows: In chapters II and III we present the source terms for

quark and gluon production from a space-time dependent chromofield. In chapter IV, we analyze the production of partons from a purely time dependent background field. Finally, we summarize and discuss our results in chapter V and conclude in chapter VI. There, one can also find a treatment on the applicability of the source terms presented in this paper. In the end, there follows an Appendix incorporating more mathematical details than were suitable for the main body of the paper.

II. SOURCE TERM FOR QUARK PRODUCTION FROM A SPACE-TIME DEPENDENT CHROMOFIELD

The amplitude for the lowest order process $A \rightarrow q(k_1)\bar{q}(k_2)$ (see Fig. (1)) contributing to the production of $q\bar{q}$ pairs from a space-time dependent classical non-abelian field $A^{a\mu}$ via vacuum-polarization is given by:

$$M =< q(k_1)\bar{q}(k_2)|S^{(1)}|0 >= \bar{u}^i(k_1)(V^F)^a_{ij\mu}A^{a\mu}(K)v^j(k_2) \tag{4}$$

with the interaction vertex:

$$(V^F)^a_{ij\mu} = ig\gamma_\mu T^a_{ij}. \tag{5}$$

Here k_1 and k_2 stand for the four momenta of the outgoing quark and antiquark respectively and $A^{a\mu}(K)$ is the Fourier transform of the space-time dependent chromofield $A^{a\mu}(x)$ with $K = k_1 + k_2$:

$$A^a_\mu(K) = \frac{1}{(2\pi)^2}\int d^4x A^a_\mu(x)\; e^{+iK\cdot x}. \tag{6}$$

$\bar{u}^i$ and v^j represent the Dirac spinors of the outgoing quark and the outgoing antiquark, respectively. Note that in the end the integrations over K have to be carried out over the correct kinematical region, because real quarks are to be produced. It is given by $(K)^2 > 4m^2$ and $K^0 > 0$. This fact is not explicitly mentioned in the formulas, but has got to be kept in mind.

In order to obtain the probability, the absolute square of the amplitude has got to be integrated over the phase space of the outgoing particles:

$$W = \sum_{spin}\int \frac{d^3k_1}{(2\pi)^3 2k_1^0}\frac{d^3k_2}{(2\pi)^3 2k_2^0}MM^*. \tag{7}$$

Putting in the definitions for the amplitudes and performing the spin-sum, as well as carrying out one of the d^4k integrations yields [14]:

$$W^{(1)}_{q\bar{q}} = \frac{g^2}{4\pi^2}\int d^4K \int \frac{d^3\vec{k}_1}{2\omega_1}\frac{d^3\vec{k}_2}{2\omega_2}\delta^{(4)}(K-k_1-k_2)\; A^a_\mu(K)A^a_\nu(-K) \\ [k_1^\mu k_2^\nu + k_2^\mu k_1^\nu - \frac{K^2}{2}g^{\mu\nu}] \tag{8}$$

where $\omega_{1,2}$ is defined as:

$$\omega_{1,2} = \sqrt{|\vec{k}_{1,2}|^2 + m^2}. \tag{9}$$

Using the Fourier transform Eq.(6) of the classical field in Eq.(8) one finds:

$$W^{(1)}_{q\bar{q}} = \frac{g^2}{(2\pi)^6} \int d^4K \int \frac{d^3\vec{k}_1}{2\omega_1} \frac{d^3\vec{k}_2}{2\omega_2} \delta^{(4)}(K - k_1 - k_2) \int d^4x_1 d^4x_2$$
$$e^{iK(x_1-x_2)} A^a_\mu(x_1) A^a_\nu(x_2) [k_1^\mu k_2^\nu + k_2^\mu k_1^\nu - \frac{K^2}{2} g^{\mu\nu}]. \tag{10}$$

From the resulting expression (10), it is then possible to extract the gauge invariant source term for the production of $q\bar{q}$ pairs. It should be mentioned here that the probability of pair production $W^{(1)}_{q\bar{q}}$ is a real quantity because T is real (see Eq. (7)). Therefore it can be shown that the imaginary part of the above expression vanishes. However, for mathematical convenience we always keep the full expression Eq. (10) and take the real part at the end of our calculations. Again, this will not be denoted in the formulas, but must not be forgotten.

Starting from Eq.(10) we find the source term for $q\bar{q}$ production:

$$\frac{dW_{q\bar{q}}}{d^4x d^3k} = \frac{g^2}{2(2\pi)^6 \omega} \; A^a_\mu(x) \int d^4x_2 \; A^a_\nu(x_2) \int \frac{d^3\vec{k}_2}{2\omega_2} \int d^4K \; \delta^{(4)}(K - k - k_2)$$
$$e^{iK(x-x_2)} [k^\mu k_2^\nu + k_2^\mu k^\nu - \frac{K^2}{2} g^{\mu\nu}]. \tag{11}$$

The d^4K integration in Eq.(11) can be carried out so that one finds:

$$\frac{dW^{(1)}_{q\bar{q}}}{d^4x d^3k} = \frac{g^2}{2(2\pi)^6 \omega} \; A^a_\mu(x) e^{ik\cdot x} \int d^4x_2 \; A^a_\nu(x_2) \; e^{-ik\cdot x_2} \int \frac{d^3\vec{k}_2}{2\omega_2}$$
$$e^{ik_2\cdot(x-x_2)} [k^\mu k_2^\nu + k_2^\mu k^\nu - (m^2 + k\cdot k_2) g^{\mu\nu}]. \tag{12}$$

The factors of k_2 in Eq.(12) can be replaced by differentiations with respect to $i(x_1 - x_2)$ which leads to:

$$\frac{dW^{(1)}_{q\bar{q}}}{d^4x d^3k} = \frac{g^2}{2(2\pi)^6 \omega} \; A^a_\mu(x) e^{ik\cdot x} \int d^4x_2 \; A^a_\nu(x_2) \; e^{-ik\cdot x_2}$$
$$[k^\mu \frac{\partial}{i\partial(x-x_2)_\nu} + \frac{\partial}{i\partial(x-x_2)_\mu} k^\nu - (m^2 + k\cdot \frac{\partial}{i\partial(x-x_2)}) g^{\mu\nu}] \int \frac{d^3\vec{k}_2}{2\omega_2} \; e^{ik_2\cdot(x-x_2)}. \tag{13}$$

The $d^3\vec{k}_2$ integration can be performed analytically which yields

$$\int \frac{d^3\vec{k}_2}{2\omega_2} \; e^{ik_2\cdot(x_1-x_2)} = 4\pi m \frac{K_1(m\sqrt{-(x_1-x_2)^2})}{\sqrt{-(x_1-x_2)^2}}. \tag{14}$$

Here $K_1(z)$ stands for the modified Bessel function of the third kind and order one. Substituting Eq.(14) into Eq.(13) and performing the differentiations [1] we obtain the general result

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[1]For the differentiation of the Bessel functions see *e.g.* Ref. [22]

$$\frac{dW_{q\bar{q}}^{(1)}}{d^4xd^3k} = \frac{g^2m}{(2\pi)^5\omega}\ A_\mu^a(x)\ e^{ik\cdot x} \int d^4x_2\ A_\nu^a(x_2)\ e^{-ik\cdot x_2}$$
$$[i(k^\mu(x-x_2)^\nu + (x-x_2)^\mu k^\nu + k\cdot(x-x_2)g^{\mu\nu})$$
$$\times(\frac{K_0(m\sqrt{-(x-x_2)^2})m\sqrt{-(x-x_2)^2} + 2K_1(m\sqrt{-(x-x_2)^2})}{[\sqrt{-(x-x_2)^2}]^3})$$
$$-m^2g^{\mu\nu}\frac{K_1(m\sqrt{-(x-x_2)^2})}{\sqrt{-(x-x_2)^2}}]. \tag{15}$$

This is the source term for the production of $q\bar{q}$ pairs from a space-time dependent chromofield. This source term contains all the information about the quark production from a space-time dependent chromofield $A^a(x)$ in the phase space of coordinate x and momentum k to the order $S^{(1)}$.

III. SOURCE TERM FOR GLUON PRODUCTION FROM A SPACE-TIME DEPENDENT CHROMOFIELD

For the gluons in QCD the generating functional is given by:

$$Z[J,\xi,\xi^\dagger] = \int [dA][d\chi][d\chi^\dagger]\exp(iS[A,\chi,\chi^\dagger] + J\cdot A + \chi\cdot\xi^\dagger + \chi^\dagger\cdot\xi). \tag{16}$$

Here J, ξ, and $\xi^\dagger$ are external sources for the gauge field A and the Faddeev-Popov ghosts fields $\chi^\dagger$ and χ, respectively. The action S is defined as follows:

$$S = \int d^4x\mathcal{L} \tag{17}$$

where $\mathcal{L}$ consists of three terms $\mathcal{L} = \mathcal{L}_G + \mathcal{L}_{GF} + \mathcal{L}_{FP}$. $\mathcal{L}_G$ is the lagrangian density of the gauge field:

$$\mathcal{L}_G = -\frac{1}{4}F_{\mu\nu}^a F^{a\mu\nu} \tag{18}$$

where $F_{\mu\nu}^a$ is the non-abelian field tensor defined as: $F_{\mu\nu}^a = \partial_\mu A_\nu - \partial_\nu A_\mu + gf^{abc}A_\mu^b A_\nu^c$ with the antisymmetric structure constant of the gauge group f^{abc} and the coupling constant g. The contribution of the gauge fixing term is given by:

$$\mathcal{L}_{GF} = -\frac{1}{2\alpha}(G^a)^2 \tag{19}$$

where G^a is a arbitrary form linear in the A-field that fixes the gauge. For the common choice $G^a = \partial^\mu A_\mu^a$ the Faddeev-Popov ghost term becomes: $\mathcal{L}_{FP} = -(\partial_\mu\chi_a^\dagger)(D^\mu\chi)_a$ where D^μ denotes the covariant derivative $D^\mu = \partial^\mu + gT^aA^{a\mu}$ with the generators of the gauge group in adjoint representation T^a. The scalar products in Eq. (16) are defined as:

$$J\cdot A = \int d^4xJ_\mu A^\mu. \tag{20}$$

In the background field method of QCD, the gauge field is split into two parts:

$$A_\mu \rightarrow A_\mu + Q_\mu. \tag{21}$$

From here on, A_μ is a classical, *i.e.* non-quantized background field and Q_μ is a quantum field representing the gluons. The lagrangian density of the gauge field now becomes a functional of A and Q:

$$\mathcal{L}_G = -\frac{1}{4} F^a_{\mu\nu}[A+Q] F^{a\mu\nu}[A+Q]. \tag{22}$$

Additionally, a special gauge is chosen. Only a gauge for Q has got to be fixed, as only this field is going to be quantized. The so called background field gauge is given by [16]:

$$G^a = \partial^\mu Q^a_\mu + g f^{abc} A^{b\mu} Q^c_\mu = (D^\mu[A] Q^a_\mu). \tag{23}$$

Due to this choice, the ghost term becomes:

$$\mathcal{L}_{FP} = -(D_\mu[A]\chi^\dagger_a)(D^\mu[A+Q]\chi^a). \tag{24}$$

The total lagrangian density $\mathcal{L}$ in the background field formalism is gauge invariant with respect to the type I gauge transformations [14]:

$$A_\mu \rightarrow A'_\mu = U A_\mu U^{-1} - \frac{i}{g}(\partial_\mu U) U^{-1}, \tag{25}$$

$$Q_\mu \rightarrow Q'_\mu = U Q_\mu U^{-1}, \tag{26}$$

and

$$\chi \rightarrow \chi' = U \chi U^{-1}, \tag{27}$$

even after quantization. In the following, all calculations are to be performed in the background field Feynman gauge, *i.e.* with $\alpha = 1$.

For the gluon pair production from a space-time dependent chromofield in the order of $S^{(1)}$ the vacuum-polarization amplitude is given by:

$$M =< k_1 k_2 | S^{(1)} | 0 > . \tag{28}$$

To this order those terms of the interaction lagrangian $\mathcal{L}_I$ involving two Q-fields are given by:

$$\begin{aligned}\mathcal{L}^{(1)}_I = & -\frac{1}{2} F^a_{\mu\nu}[A] g f^{abc} Q^{b\mu} Q^{c\nu} - \frac{1}{2}(\partial_\mu Q^a_\nu - \partial_\nu Q^a_\mu) g f^{abc}(A^{b\mu} Q^{c\nu} + Q^{b\mu} A^{c\nu}) \\ & -\frac{1}{4} g^2 f^{abc} f^{ab'c'} (A^b_\mu Q^c_\nu + Q^b_\mu A^c_\nu)(A^{b'\mu} Q^{c'\nu} + Q^{b'\mu} A^{c'\nu}) \\ & -\partial_\lambda Q^{a\lambda} g f^{abc} A^b_\kappa Q^{c\kappa} - \frac{1}{2} g^2 f^{abc} f^{ab'c'} A^b_\lambda Q^{c\lambda} A^{b'}_\kappa Q^{c'\kappa}.\end{aligned} \tag{29}$$

From these terms one can read off the necessary Feynman rules. For the production of two gluons by coupling to the field once (see Fig.(2a)), one obtains the vertex:

$$(V_{1A})^{abd}_{\mu\nu\rho} = g f^{abd}[-2g_{\mu\rho}K_\nu - g_{\nu\rho}(k_1 - k_2)_\mu + 2g_{\mu\nu}K_\rho], \tag{30}$$

where $K = k_1 + k_2$. If two couplings to the field are involved (see Fig.(2b)), one receives:

$$\begin{aligned}(V_{2A})^{abcd}_{\mu\nu\lambda\rho} = -ig^2[&f^{abx}f^{xcd}(g_{\mu\lambda}g_{\nu\rho} - g_{\mu\rho}g_{\nu\lambda} + g_{\mu\nu}g_{\lambda\rho}) \\ &+f^{adx}f^{xbc}(g_{\mu\nu}g_{\lambda\rho} - g_{\mu\lambda}g_{\nu\rho} - g_{\mu\rho}g_{\nu\lambda}) \\ &+f^{acx}f^{xbd}(g_{\mu\nu}g_{\lambda\rho} - g_{\mu\rho}g_{\nu\lambda})].\end{aligned} \tag{31}$$

The above rules coincide with those derived in [23]. The total amplitude for the production of gluon pairs is now equal to the sum of the single contributions:

$$M = M_{1A} + M_{2A}. \tag{32}$$

If the correct weight factors are included, so that the lagrangian density can again be retrieved, one finds for the first term:

$$M_{1A} = \frac{(2\pi)^2}{2} \int d^4K \delta^{(4)}(K - k_1 - k_2) A^{a\mu}(K) \epsilon^{b\nu}(k_1) \epsilon^{d\rho}(k_2) (V_{1A})^{abd}_{\mu\nu\rho} \tag{33}$$

where finally the allowed kinematical region of integration is given by $(K)^2 > 0$ and $K^0 > 0$. The second term is given by:

$$M_{2A} = \frac{1}{4} \int d^4k_3 d^4k_4 \delta^{(4)}(k_1 + k_2 - k_3 - k_4) A^{a\mu}(k_3) A^{c\lambda}(k_4) \epsilon^{b\nu}(k_1) \epsilon^{d\rho}(k_2) (V_{2A})^{abcd}_{\mu\nu\lambda\rho}, \tag{34}$$

where the limitations $(k_3 + k_4)^2 > 0$ and $k_3^0 + k_4^0 > 0$ have to be obeyed in the end. The probability can again be obtained by making use of Eq.(7). In order to obtain the physical polarization of the gluons, we use the following spin-sums:

$$\sum_{spin} \epsilon^\nu(k_1)\epsilon^{*\nu'}(k_1) = \sum_{spin} \epsilon^\nu(k_2)\epsilon^{*\nu'}(k_2) = -g^{\nu\nu'}, \tag{35}$$

but later on deduct the corresponding probabilities for the processes involving ghost instead of gluons as shown in Fig.(3). In our case, the total probability for the production of gluons not yet corrected for ghosts consists of four terms which add up:

$$W^A = W_{1A,1A} + W_{1A,2A} + W_{2A,1A} + W_{2A,2A}. \tag{36}$$

By definition, we have for the first term in Eq. (36):

$$W_{1A,1A} = \frac{1}{2} \sum_{spin} \int \frac{d^3k_1}{(2\pi)^3 2k_1^0} \frac{d^3k_2}{(2\pi)^3 2k_2^0} M_{1A} M_{1A}^*. \tag{37}$$

For the integration over the phase-space of the gluons there is an additional factor of 1/2 because the particles in the final state are identical. The introduction of the Fourier transform Eq.(6) and the application of

$$f^{abd}f^{a'bd} = 3\delta^{aa'} \tag{38}$$

yields:

$$W_{1A,1A} = \frac{3g^2}{8(2\pi)^6}\int \frac{d^3k_1}{k_1^0}\frac{d^3k_2}{k_2^0}d^4x d^4x' e^{i(k_1+k_2)\cdot(x-x')}A^{a\mu}(x)A^{a\mu'}(x')$$
$$[2g_{\mu\mu'}(k_1+k_2)^2 - 2(k_1+k_2)_\mu(k_1+k_2)_{\mu'} + (k_1-k_2)_\mu(k_1-k_2)_{\mu'}]. \tag{39}$$

Again, it has to be noted that the real part of this quantity has to be taken. By the same means as in the quark case, we obtain the first contribution to the gluon source term:

$$\frac{dW_{1A,1A}}{d^4xd^3k} = \frac{3g^2}{8(2\pi)^6}\int \frac{d^3k_2}{k^0k_2^0}d^4x' e^{i(k+k_2)\cdot(x-x')}A^{a\mu}(x)A^{a\mu'}(x')$$
$$[2g_{\mu\mu'}(k+k_2)^2 - 2(k+k_2)_\mu(k+k_2)_{\mu'} + (k-k_2)_\mu(k-k_2)_{\mu'}]. \tag{40}$$

Making use of the following relation

$$\int \frac{d^3k_2}{|\vec{k}_2|}e^{ik_2\cdot(x-x')} = -\frac{8\pi}{(x-x')^2}, \tag{41}$$

and explicitly calculating the derivatives afterwards results in the final form (see Appendix):

$$\frac{dW_{1A,1A}}{d^4xd^3k} = \frac{3g^2}{2(2\pi)^5k^0}\int d^4x' e^{ik\cdot(x-x')}A^{a\mu}(x)A^{a\mu'}(x')[k_\mu k_{\mu'} - 4g_{\mu\mu'}k^\nu i\frac{(x-x')_\nu}{(x-x')^2}$$
$$+3(k_\mu i\frac{(x-x')_{\mu'}}{(x-x')^2} + k_{\mu'}i\frac{(x-x')_\mu}{(x-x')^2}) + 2\frac{g_{\mu\mu'}}{(x-x')^2} - 4\frac{(x-x')_\mu(x-x')_{\mu'}}{(x-x')^4}]\frac{1}{(x-x')^2}. \tag{42}$$

Now we perform the same procedure with the other contributions. First for the two mixed contributions we get:

$$W_{1A,2A} = W_{2A,1A} = \frac{1}{2}\sum_{spin}\int \frac{d^3k_1}{(2\pi)^3 2k_1^0}\frac{d^3k_2}{(2\pi)^3 2k_2^0}M_{2A}M_{1A}^*. \tag{43}$$

After expressing the probability in terms of the Fourier transforms of the A-fields:

$$W_{1A,2A} = W_{2A,1A} =$$
$$\frac{-ig^3}{64(2\pi)^{10}}\int \frac{d^3k_1}{k_1^0}\frac{d^3k_2}{k_2^0}d^4k_3d^4k_4d^4Kd^4x_3d^4x_4d^4x\delta^{(4)}(K-k_3-k_4)\delta^{(4)}(K-k_1-k_2)$$
$$e^{i[k_3\cdot x_3+k_4\cdot x_4-K\cdot x]}A^{a\mu}(x_3)A^{c\lambda}(x_4)A^{a'\mu'}(x)24f^{a'ac}K_\lambda g_{\mu\mu'}, \tag{44}$$

we can extract the source term:

$$\frac{dW_{2A,1A}}{d^4xd^3k} = \frac{-3ig^3}{8(2\pi)^{10}}\int \frac{d^3k_2}{k^0k_2^0}d^4k_3d^4k_4d^4Kd^4x_3d^4x_4\delta^{(4)}(K-k_3-k_4)\delta^{(4)}(K-k-k_2)$$
$$e^{i[k_3\cdot x_3+k_4\cdot x_4-K\cdot x]}A^{a\mu}(x_3)A^{c\lambda}(x_4)A^{a'\mu'}(x)f^{a'ac}K_\lambda g_{\mu\mu'}. \tag{45}$$

At this point, we again need Eq. (41) to find (see Appendix):

$$\frac{dW_{2A,1A}}{d^4xd^3k} = \frac{3ig^3}{2(2\pi)^5k^0}\int d^4x'e^{ik\cdot(x'-x)}A^{c\lambda}(x')(A^a(x')\cdot A^{a'}(x))f^{a'ac}(\frac{k_\lambda}{(x'-x)^2}+2i\frac{(x'-x)_\lambda}{(x'-x)^4}). \quad (46)$$

And finally for the last term: From the definition

$$W_{2A,2A} = \frac{1}{2}\sum_{spin}\int\frac{d^3k_1}{(2\pi)^32k_1^0}\frac{d^3k_2}{(2\pi)^32k_2^0}M_{2A}M_{2A}^* \quad (47)$$

we obtain

$$\begin{aligned} W_{2A,2A} = {} & \frac{g^4}{64(2\pi)^{14}}\int\frac{d^3k_1}{k_1^0}\frac{d^3k_2}{k_2^0}d^4k_3d^4k_4d^4k_3'd^4k_4'd^4x_3d^4x_4d^4x_3'd^4x_4'e^{i[k_3\cdot x_3+k_4\cdot x_4-k_3'\cdot x_3'-k_4'\cdot x_4']} \\ & \delta^{(4)}(k_1+k_2-k_3-k_4)\delta^{(4)}(k_3+k_4-k_3'-k_4')A^{a\mu}(x_3)A^{c\lambda}(x_4)A^{a'\mu'}(x_3')A^{c'\lambda'}(x_4') \\ & [2g_{\mu\lambda}g_{\mu'\lambda'}(f^{abx}f^{xcd}+f^{adx}f^{xcb})(f^{a'bx'}f^{x'c'd}+f^{a'dx'}f^{x'c'b})+24g_{\mu\mu'}g_{\lambda\lambda'}f^{acx}f^{a'c'x}]. \quad (48)\end{aligned}$$

And from there the source term:

$$\begin{aligned} \frac{dW_{2A,2A}}{d^4xd^3k} = {} & \frac{g^4}{32(2\pi)^{14}}\int\frac{d^3k_2}{k^0k_2^0}d^4k_3d^4k_4d^4k_3'd^4k_4'd^4x_4d^4x_3'd^4x_4'e^{i[k_3\cdot x+k_4\cdot x_4-k_3'\cdot x_3'-k_4'\cdot x_4']} \\ & \delta^{(4)}(k+k_2-k_3-k_4)\delta^{(4)}(k_3+k_4-k_3'-k_4')A^{a\mu}(x)A^{c\lambda}(x_4)A^{a'\mu'}(x_3')A^{c'\lambda'}(x_4') \\ & [g_{\mu\lambda}g_{\mu'\lambda'}(f^{abx}f^{xcd}+f^{adx}f^{xcb})(f^{a'bx'}f^{x'c'd}+f^{a'dx'}f^{x'c'b})+12g_{\mu\mu'}g_{\lambda\lambda'}f^{acx}f^{a'c'x}]. \quad (49)\end{aligned}$$

Here we apply Eq. (41) to receive (see Appendix)

$$\begin{aligned} \frac{dW_{2A,2A}}{d^4xd^3k} = {} & \frac{-g^4}{8(2\pi)^5k^0}\int d^4x_3'e^{ik\cdot(x-x_3')}\frac{1}{(x-x_3')^2}A^{a\mu}(x)A^{c\lambda}(x)A^{a'\mu'}(x_3')A^{c'\lambda'}(x_3') \\ & [g_{\mu\lambda}g_{\mu'\lambda'}(f^{abx}f^{xcd}+f^{adx}f^{xcb})(f^{a'bx'}f^{x'c'd}+f^{a'dx'}f^{x'c'b})+12g_{\mu\mu'}g_{\lambda\lambda'}f^{acx}f^{a'c'x}]. \quad (50)\end{aligned}$$

As already remarked before, we also have to calculate the corresponding terms for the ghosts in order to be able to remove the contributions by unphysical polarizations of the gluons. To the ghost matrix element, there contribute the two Feynman diagrams shown in Fig. (3) with the interaction vertices

$$(V_{1A}^{FP})_\mu^{abd} = +gf^{abd}(k_1-k_2)_\mu, \quad (51)$$

and

$$(V_{2A}^{FP})_{\mu\lambda}^{abcd} = -ig^2g_{\mu\lambda}(f^{abx}f^{xcd}+f^{adx}f^{xcb}). \quad (52)$$

These vertices are obtained from the ghost lagrangian density:

$$\mathcal{L}_{FP}^{(I)} = -(\partial_\mu\chi^{a\dagger})f^{abc}A_\mu^b\chi^c - (f^{abc}A_\mu^b[A]\chi^{c\dagger})\partial_\mu\chi^a - (f^{abc}A_\mu^b\chi^{c\dagger})f^{ade}A_\mu^d\chi^e \quad (53)$$

Including the correct weight factors we obtain the amplitude:

$$(M^{FP})^{bd} = (M_{1A}^{FP})^{bd} + (M_{2A}^{FP})^{bd} \quad (54)$$

with

$$(M^{FP}_{1A})^{bd} = \frac{(2\pi)^2}{2}\int d^4K\delta^{(4)}(k_1+k_2-K)A^{a\mu}(K)(V^{FP}_{1A})^{abd}_{\mu} \tag{55}$$

and

$$(M^{FP}_{2A})^{bd} = \frac{1}{4}\int d^4k_3 d^4k_4\delta^{(4)}(k_1+k_2-k_3-k_4)A^{a\mu}(k_3)A^{c\lambda}(k_4)(V^{FP}_{2A})^{abcd}_{\mu\lambda}. \tag{56}$$

The probability is defined as for the gluons but without the factor 1/2 for the phase space:

$$W^{FP} = \int \frac{d^3k_1}{(2\pi)^3 2k_1^0}\frac{d^3k_2}{(2\pi)^3 2k_2^0}(M^{FP})^{bd}(M^{FP})^{*bd}. \tag{57}$$

This time, there are only two contributions, because the cross-terms vanish:

$$W^{FP} = W^{FP}_{1A,1A} + W^{FP}_{2A,2A}. \tag{58}$$

Starting from the definition of the first term

$$W^{FP}_{1A,1A} = \int \frac{d^3k_1}{(2\pi)^3 2k_1^0}\frac{d^3k_2}{(2\pi)^3 2k_2^0}(M^{FP}_{1A})^{bd}(M^{FP}_{1A})^{*bd}, \tag{59}$$

we carry out the same steps as for the derivation of Eq. (42) from Eq. (37) through Eq.(39). From

$$W^{FP}_{1A,1A} = \frac{3g^2}{16(2\pi)^2}\int \frac{d^3k_1}{k_1^0}\frac{d^3k_2}{k_2^0}d^4K\delta^{(4)}(k_1+k_2-K)A^{a\mu}(K)A^{*a\mu'}(K)(k_1-k_2)_\mu(k_1-k_2)_{\mu'} \tag{60}$$

we extract the source term:

$$\frac{dW^{FP}_{1A,1A}}{d^4x d^3k} = \frac{3g^2}{4(2\pi)^5 k^0}\int d^4x' e^{ik\cdot(x-x')}A^{a\mu}(x)A^{a\mu'}(x')$$
$$[k_\mu i\frac{(x-x')_{\mu'}}{(x-x')^2} + k_{\mu'} i\frac{(x-x')_\mu}{(x-x')^2} - k_\mu k_{\mu'} - 2\frac{g_{\mu\mu'}}{(x-x')^2} + 4\frac{(x-x')_\mu (x-x')_{\mu'}}{(x-x')^4}]\frac{1}{(x-x')^2}. \tag{61}$$

Analogously, we obtain the second contribution from its definition:

$$W^{FP}_{2A,2A} = \int \frac{d^3k_1}{(2\pi)^3 2k_1^0}\frac{d^3k_2}{(2\pi)^3 2k_2^0}(M^{FP}_{2A})^{bd}(M^{FP}_{2A})^{*bd} \tag{62}$$

and finally get:

$$W^{FP}_{2A,2A} = \frac{g^4}{64(2\pi)^6}\int \frac{d^3k_1}{k_1^0}\frac{d^3k_2}{k_2^0}d^4k_3 d^4k_4 d^4k'_3 d^4k'_4\delta^{(4)}(k_1+k_2-k_3-k_4)\delta^{(4)}(k_1+k_2-k'_3-k'_4)$$
$$A^{a\mu}(k_3)A^{c\lambda}(k_4)A^{*a'\mu'}(k'_3)A^{*c'\lambda'}(k'_4)g_{\mu\lambda}g_{\mu'\lambda'}(f^{abx}f^{xcd}+f^{adx}f^{xcb})(f^{a'bx'}f^{x'c'd}+f^{a'dx'}f^{x'c'b}). \tag{63}$$

Comparison to Eq.(50) yields the source term:

$$\frac{dW^{FP}_{2A,2A}}{d^4x d^3k} = \frac{-g^4}{16(2\pi)^5 k^0}\int d^4x'_3 e^{ik\cdot(x-x'_3)}\frac{1}{(x-x'_3)^2}A^{a\mu}(x)A^{c\lambda}(x)A^{a'\mu'}(x'_3)A^{c'\lambda'}(x'_3)$$
$$g_{\mu\lambda}g_{\mu'\lambda'}(f^{abx}f^{xcd}+f^{adx}f^{xbc})(f^{a'bx'}f^{x'c'd}+f^{a'dx'}f^{x'bc'}). \tag{64}$$

In the end, we obtain the required source term for the production of gluon pairs from a space-time dependent chromofield via vacuum-polarization by combining all the contributions in the following manner:

$$W_{gg} = W_{1A,1A} - W^{FP}_{1A,1A} + 2W_{2A,1A} + W_{2A,2A} - W^{FP}_{2A,2A}. \tag{65}$$

So, we receive:

$$\begin{aligned}\frac{dW_{gg}}{d^4xd^3k} = \frac{1}{(2\pi)^5k^0}\int d^4x' e^{ik\cdot(x-x')}\frac{1}{(x-x')^2}\{\frac{3}{4}g^2A^{a\mu}(x)A^{a\mu'}(x')[3k_\mu k_{\mu'} - 8g_{\mu\mu'}k^\nu i\frac{(x-x')_\nu}{(x-x')^2} \\ +5(k_\mu i\frac{(x-x')_{\mu'}}{(x-x')^2} + k_{\mu'}i\frac{(x-x')_\mu}{(x-x')^2}) + 6\frac{g_{\mu\mu'}}{(x-x')^2} - 12\frac{(x-x')_\mu(x-x')_{\mu'}}{(x-x')^4}] \\ -3ig^3A^{a\mu}(x')A^{c\lambda}(x')A^{a'\mu'}(x)f^{a'ac}K_\lambda g_{\mu\mu'} \\ -\frac{1}{16}g^4A^{a\mu}(x)A^{c\lambda}(x)A^{a'\mu'}(x')A^{c'\lambda'}(x') \\ [g_{\mu\lambda}g_{\mu'\lambda'}(f^{abx}f^{xcd} + f^{adx}f^{xcb})(f^{a'bx'}f^{x'c'd} + f^{a'dx'}f^{x'c'b}) + 24g_{\mu\mu'}g_{\lambda\lambda'}f^{acx}f^{a'c'x}]\}. \end{aligned} \tag{66}$$

This is the source term for the production of gluon pairs for any arbitrary space-time dependent chromofield in the order $S^{(1)}$.

It can be checked that this source term is gauge invariant with respect to type-(I)-gauge transformations in the following manner (see also [25]). One can observe that the gauge invariant part of the lagrangian which in each term contains two Q fields or two ghost fields is given by:

$$\begin{aligned}\mathcal{L} = -\frac{1}{2}gf^{abc}\ F^a_{\mu\nu}[A]Q^{b\mu}Q^{c\nu} \\ -\frac{1}{4}[D_\mu[A]Q^{a\nu} - D_\nu[A]Q^{a\mu}][D_\mu[A]Q^{a\nu} - D_\nu[A]Q^{a\mu}] \\ -\frac{1}{2\alpha}[D_\mu[A]Q^{a\mu}]^2 - (D_\mu[A]\chi^\dagger_a)D_\mu[A]\chi^a.\end{aligned} \tag{67}$$

This lagrangian density is gauge invariant with respect to the gauge transformations in Eqs.(25), (26), and (27). It can be checked that the free terms, *i.e.* terms not involving A, of the gauge invariant lagrangian density (Eq. (67)), when evaluated between an initial vacuum state $|0>$ and a final physical two gluon state $< k_1, k_2|$, do not contribute to the probability. Hence, the expression we have used for $\mathcal{L}$ in Eq. (29) together with Eq. (53) for the ghosts gives the correct result for the gluon pair production probability, invariant with respect to the above gauge transformations.

IV. SOURCE TERM FOR PARTON PRODUCTION FROM A PURELY TIME DEPENDENT CHROMOFIELD

Our main purpose is to use the obtained source terms of quarks and gluons in the relativistic non-abelian transport equations to study the production and equilibration of the quark-gluon plasma expected to be formed in ultra relativistic heavy-ion collisions (URHIC). Solving the transport equations with all the effects, such as collisions among the partons,

acceleration of the partons by the background field, precession of the color charge in the group space and production of the partons from the background field involves much more numerical work as was done in [24]. In the following we are going to analyze an example: We will consider a time dependent chromofield and examine the behavior of the source term in the phase-space. Solutions of the transport equations including these source terms to study the production and equilibration of quark-gluon plasma will be presented elsewhere.

A. Source Terms for any Arbitrary Time Dependent Chromofield

For a purely time-dependent field Eq. (11) reduces to:

$$\frac{dW_{q\bar{q}}}{d^4xd^3k} = \frac{g^2}{2(2\pi)^3\omega}\; A^a_\mu(t) \int dt_2\; A^a_\nu(t_2) \int \frac{d^3\vec{k}_2}{2\omega_2} \int d^4K\; \delta^{(4)}(K-k-k_2)\; \delta^{(3)}(\vec{K}) \\ e^{i(K\cdot x - K^0 t_2)}[k^\mu k_2^\nu + k_2^\mu k^\nu - \frac{K^2}{2} g^{\mu\nu}]. \tag{68}$$

After integrating over d^4K and $d^3\vec{k}_2$ and taking care of the Dirac δ factors we find

$$\frac{dW_{q\bar{q}}}{d^4xd^3k} = \frac{g^2}{2(2\pi)^3\omega} \int dt_2 \frac{e^{2i\omega(t-t_2)}}{2\omega}[2\omega^2(\vec{A}^a(t)\cdot\vec{A}^a(t_2)) - 2(\vec{A}^a(t)\cdot\vec{k})(\vec{A}^a(t_2)\cdot\vec{k})]. \tag{69}$$

Here, we have presented Eq. (69) in the form of scalar products of three-vectors. The last remaining time integration then constitutes a Fourier transform from t_2 to $2\omega_1$. The final result for the source term in the case of a time-dependent chromofield is

$$\frac{dW_{q\bar{q}}}{d^4xd^3k} = \frac{g^2\sqrt{2\pi}}{2(2\pi)^3\omega^2} e^{2i\omega t}[\omega^2(\vec{A}^a(t)\cdot\vec{A}^{*a}(2\omega)) - (\vec{A}^a(t)\cdot\vec{k})(\vec{A}^{*a}(2\omega)\cdot\vec{k})]. \tag{70}$$

Analogously, the Eqs. (40,45, and 49) for the gluons and the Eqs. (61 and 64) for the ghosts simplify to (see Appendix):

$$\frac{dW_{1A,1A}}{d^4xd^3k} = \frac{3g^2\sqrt{2\pi}}{2(2\pi)^3} e^{2ik^0t}\; [(\vec{A}^a(t)\cdot\frac{\vec{k}}{k^0})(\vec{A}^{*a}(2k^0)\cdot\frac{\vec{k}}{k^0}) - 2(\vec{A}^a(t)\cdot\vec{A}^{*a}(2k^0))], \tag{71}$$

$$\frac{dW_{2A,1A}}{d^4xd^3k} = \frac{-3ig^3}{4(2\pi)^3k^0} \int dt_3 e^{2ik^0(t_3-t)} A^{a\mu}(t_3) A^{c0}(t_3) A^{a'\mu'}(t) f^{a'ac} g_{\mu\mu'}, \tag{72}$$

$$\frac{dW_{2A,2A}}{d^4xd^3k} = \frac{g^4}{32(2\pi)^3(k^0)^2} \int dt'_3 e^{2ik^0(t-t'_3)} A^{a\mu}(t) A^{c\lambda}(t) A^{a'\mu'}(t'_3) A^{c'\lambda'}(t'_3) \\ [g_{\mu\lambda} g_{\mu'\lambda'}(f^{abx}f^{xcd} + f^{adx}f^{xcb})(f^{a'bx'}f^{x'c'd} + f^{a'dx'}f^{x'c'b}) + 12 g_{\mu\mu'} g_{\lambda\lambda'} f^{acx} f^{a'c'x}], \tag{73}$$

$$\frac{dW^{FP}_{1A,1A}}{d^4xd^3k} = \frac{3g^2\sqrt{2\pi}}{4(2\pi)^3} e^{2ik^0t} (\vec{A}^a(t)\cdot\frac{\vec{k}}{k^0})(\vec{A}^{*a}(2k^0)\cdot\frac{\vec{k}}{k^0}), \tag{74}$$

and

$$\frac{dW^{FP}_{2A,2A}}{d^4xd^3k} = \frac{g^4}{64(2\pi)^3(k^0)^2}\int dt_3' e^{2ik^0(t-t_3')} A^{a\mu}(t)A^{c\lambda}(t)A^{a'\mu'}(t_3')A^{c'\lambda'}(t_3')$$
$$g_{\mu\lambda}g_{\mu'\lambda'}(f^{abx}f^{xcd}+f^{adx}f^{xcb})(f^{a'bx'}f^{x'c'd}+f^{a'dx'}f^{x'c'b}). \tag{75}$$

Adding the gluon and deducting the ghost contributions, we obtain:

$$\frac{dW_{gg}}{d^4xd^3k} = e^{2ik^0t}\frac{\sqrt{2\pi}}{(2\pi)^3}\{\frac{3g^2}{4}\ [(\vec{A}^a(t)\cdot\frac{\vec{k}}{k^0})(\vec{A}^{*a}(2k^0)\cdot\frac{\vec{k}}{k^0}) - 4(\vec{A}^a(t)\cdot\vec{A}^{*a}(2k^0))]$$
$$+\frac{6ig^3}{4k^0}(A^{a\mu}*A^{c0})^*(2k^0)A^{a'\mu'}(t)f^{a'ac}g_{\mu\mu'}$$
$$+\frac{g^4}{64(k^0)^2}A^{a\mu}(t)A^{c\lambda}(t)(A^{a'\mu'}*A^{c'\lambda'})^*(2k^0)$$
$$[g_{\mu\lambda}g_{\mu'\lambda'}(f^{abx}f^{xcd}+f^{adx}f^{xcb})(f^{a'bx'}f^{x'c'd}+f^{a'dx'}f^{x'c'b})+24g_{\mu\mu'}g_{\lambda\lambda'}f^{acx}f^{a'c'x}]\}. \tag{76}$$

This is the general source term for the production of gluon pairs in the presence of a purely time dependent chromofield.

B. A Special Case

We now choose a special form of the field in order to get some insight into the behavior of the obtained source terms for quarks and gluons. For simplicity we choose the field to be

$$A^{a3}(t) = A_{in}e^{-|t|/t_0},\ t_0>0,\ a=1,...,8, \tag{77}$$

and all other components are equal to zero. Many other forms could have been taken. We have chosen this option just to get a feeling for how the source term in the phase-space behaves. This choice is also inspired from the numerical study [24] which shows that the decay of the field is close to this behavior. In any case the actual form of the decay of the classical field can only be determined from a self consistent solution of the relativistic non-abelian transport equations, as the transport equations have to be combined with the energy momentum conservation equations for particles and fields. That is why the exact form for the decay of the fields (due to the production, acceleration of partons, precession of color, collision among partons, and expansion of the system) can be determined from self consistent transport studies (see [24] for details). We will use the general source terms (see section III and IV) in the relativistic transport equations to describe the production and equilibration of the quark-gluon plasma in ultra relativistic heavy-ion collisions in the future.

The Fourier transform of Eq. (77) is given by

$$A^{a3}(-2|\vec{k}|) = A_{in}\frac{2}{\sqrt{2\pi}}\frac{t_0}{1+4|\vec{k}|^2t_0^2}. \tag{78}$$

Putting Eqs. (77) and (78) into Eq.(70) and afterwards summing over color space yields:

$$\frac{dW_{q\bar{q}}}{d^4xd^3k} = 16\frac{\alpha_S}{(2\pi)^2}(A_{in})^2 e^{2i\omega t}e^{-|t|/t_0}\frac{t_0}{1+4\omega^2 t_0^2}\frac{m_T^2}{\omega^2}, \tag{79}$$

with $m_T^2 = m^2 + k_T^2$ and where k_T is the transverse momentum. Using, in the same way, Eqs. (77) and (78) in Eq.(76) results in:

$$\begin{aligned}\frac{dW_{gg}}{d^4xd^3k} = &\frac{24\alpha_S}{(2\pi)^2}(A_{in})^2 e^{2ik^0 t}e^{-|t|/t_0}\frac{t_0}{1+4(k^0)^2 t_0^2}(-3-\frac{k_T^2}{(k^0)^2}) \\ &+\frac{36\alpha_S^2}{2\pi}(A_{in})^4 e^{2ik^0 t}e^{-2|t|/t_0}\frac{t_0}{1+(k^0)^2 t_0^2}\frac{1}{(k^0)^2}.\end{aligned} \tag{80}$$

The contribution of the mixed term (46) vanishes for all fields of the form $A^{a\mu}(x) = A_{in}^{a\mu} f(x)$. For our choice the contribution of the last term in Eq. (50) is zero.

In the next chapter, we will present plots for the source terms derived above (Eqs. (79) and (80)) for our choice of input parameters.

V. RESULTS AND DISCUSSION

The general results for the source terms for the production of quark-antiquark pairs and gluon pairs by vacuum polarization in the presence of a general classical space-time dependent chromofield to the order $S^{(1)}$ are given by Eq.(15) and Eq.(66), respectively. Continuing from this point, *i.e.* using these source terms in relativistic non-abelian transport equations to study the production and equilibration of the quark-gluon plasma, involves extensive numerical computations [24] which have to be represented later on. In order to get an idea about the behavior of the source terms, we derived the general expressions for them in the presence of a purely time-dependent field. Subsequently, we assumed a certain, physically motivated form for the background field, given in Eq. (77) to obtain Eqs. (79) and (80). We now present the results from the above equations.

We relate the energy k_0 of the produced particles to the rapidity y by:

$$k^0 = k_T \cosh(y), \tag{81}$$

where k_T is the transverse momentum. If not stated otherwise, we choose the following parameters: $\alpha_S = 0.15$, $A_{in} = 1.5GeV$, $k_T = 1.5GeV$, $y = 0$, and $t_0 = 0.5fm$ and the quarks are considered to be massless. These values might not correspond to an exact combination of values in an URHIC. We have just chosen a set of values as an example in order to demonstrate the properties of our source term.

In Fig. (4), we plot the source term

$$S = \frac{dW}{d^4xd^3k}. \tag{82}$$

for quarks (Eq. (79)) and for gluons (Eq. (80)) versus time for the above choice of parameters. All quark graphs are to be multiplied by a factor of two. The exponential decay originates mostly from the decay of the model-field. Of course, every decaying field will transfer this quality to the production rate, but this is not necessarily an exponential decrease. The oscillatory behavior is due to the exponential factor with imaginary exponent.

This factor is already present in the general formula. Physically, it seems to indicate that there exist periods of particle creation and particle annihilation which follow each other periodically. Additionally, the frequency of this oscillation is varying with the energy of the produced particles. This oscillatory behavior of the source term will play a crucial role once it is included in a self consistent transport calculation. It can also be seen in the Figure that there are considerably more gluons produced than quarks.

In order to get a better overall understanding, we present in the following two Figs. ((5) and (6)) the source term integrated over positive times, *i.e*:

$$T = \frac{dW}{2d^3xd^3k}. \tag{83}$$

The time-integrated source terms can be regarded as a measure for the net-production of particles in an infinitesimal volume around any given point in the phase-space. It does not show the oscillatory behavior which gives a totally different picture for different times. For our choice of the field (Eq. (77)) T takes the form:

$$\frac{dW_{q\bar{q}}}{2d^3xd^3k} = 16\frac{\alpha_S}{(2\pi)^2}(A_{in})^2(\frac{t_0}{1+4(k_T)^2\cosh^2(y)t_0^2})^2\frac{1}{\cosh^2(y)} \tag{84}$$

and

$$\begin{aligned}\frac{dW_{gg}}{2d^3xd^3k} = 3\frac{\alpha_S}{(2\pi)^4}(A_{in})^2(-16(\frac{t_0}{1+4(k_T)^2\cosh^2(y)t_0^2})^2[3+\frac{1}{\cosh^2(y)}] \\ +3(A_{in})^2\frac{g^2}{(k_T)^2\cosh^2(y)}(\frac{t_0}{1+(k_T)^2\cosh^2(y)t_0^2})^2)\end{aligned} \tag{85}$$

respectively.

The decay behavior with the transverse momentum observed in Fig. (5) is mostly due to the choice of the field. Only in the second contribution to the gluon source term there is already a factor $1/(k^0)^2$ present in the general formula. Choosing e.g. a Gaussian form for the field, would also have resulted in a Gaussian decrease of the production rate with the transverse momentum of the particles. As the momentum structure of the general equations is mostly based on the k^0-component, the origin for the typical rapidity behavior is mainly the same as for the behavior for changing transverse momentum, see Fig. (6).

In the last plot Fig. (7), we plot the ratio of the source terms for quark-antiquark pairs and gluon pairs:

$$R = \frac{dW_{q\bar{q}}}{2d^3xd^3k}/\frac{dW_{gg}}{d^3xd^3k}. \tag{86}$$

It can now be seen directly from Fig. (7) that there are less quarks produced than gluons. We observe for this model field that a stronger coupling, a stronger chromofield and/or a slowlier varying field emphasize the gluon-pair production even more in comparison to the production of $q\bar{q}$ pairs.

VI. CONCLUSION

We have derived the source term for quark-antiquark and gluon pair production via vacuum polarization in the presence of a space-time dependent chromofield in the order $S^{(1)}$. We have used the background field method of QCD for this purpose. The production of quark-antiquark pairs from a Yang-Mills field is very similar to the production of electron-positron pairs from a Maxwell field in QED. To the order $S^{(1)}$ the source term for gluon pairs in the presence of a general space-time dependent chromofield consists of three contributions, each of a different order in the coupling constant g.

To obtain an insight into the source terms, we have derived them for a purely time dependent chromofield and have presented the results for an exponentially decaying field. In comparison to the arbitrary space-time dependent field, the equations obtained in this case are considerably simpler but still too complicated as to interpret them at first sight. For this reason, not for giving a realistic physical prediction, we evaluated the source term in the purely time-dependent case for a given form of the chromofield. It was chosen, so that analytical solutions were possible, but still it is motivated by physical arguments. The results thus obtained have been plotted and discussed.

We observe that the source term not only creates partons from the field but also destroys them to produce field. One can recognize periodic fluctuations of the production rate with time which become more rapid for higher particle energies. This behavior is inherent to the general time-dependent formulas and is not connected to a special choice of the chromofield. This feature may play an important role in the evolution of the quark-gluon plasma. The mere existence of the field might complicate the transport studies. For example, one usually solves the hydrodynamic evolution equation [26] when the plasma is in local thermal equilibrium. The presence of the chromofield results in the necessity to solve chromohydrodynamic evolution equations, thus one needs to know how fast the field decays. The description of all processes will become substantially easier if the field strength is already negligible before equilibration sets in. In all plots it can be seen that there are considerably more gluon pairs than quark-antiquark pairs produced by this mechanism.

Finally, we discuss the range of validity of the source terms derived in this paper. Being perturbative, our result is only applicable in situations, where certain conditions are satisfied. Firstly, in the framework of our approach we can merely properly describe the production of partons with momentum p larger than the field strength A multiplied by the coupling constant g. This means, if the field is very strong and gA is truely large, our calculation is not valid, even if the coupling g is weak. This is similarly true for the Schwinger mechanism for pair production in QED in the presence of a weak field (see Eq. 6.33 in [12]). Although QED is a weak coupling theory the perturbative calculation of Schwinger is applicable only in the limit of small eA. However, for a very small coupling, a truely large background field has got to be present before this condition is violated for reasonable momenta p. Secondly, for the perturbative method to be applicable gA has got to be larger than the non-perturbative scale Λ_{QCD}. For the condition $gA < \Lambda_{QCD}$ the perturbative calculation is not valid and parton production has to be computed via non-perturbative methods, which is unfortunately non-trivial. So, the region in which our perturbative approach is applicable is given by : $p > gA > \Lambda_{QCD}$.

Let us consider a realistic situation in a heavy-ion collision at LHC (Pb-Pb at $\sqrt{s}$= 5.5

TeV). For the energy density formed in this experiment one expects $\epsilon \sim 1000$ GeV/fm^3 [27,28,18] and for the strong coupling constant $\alpha_s \sim 0.15$ [27]. Assuming that all the energy density ϵ is deposited in the field sector we can get a rough estimate for gA ($E \sim \sqrt{2\epsilon}$ and $gA^2 \sim E$) to be: $gA \sim 2$ GeV. Hence, initially, a perturbative treatment might be applicable for the production of partons whose momentum is greater than 2 GeV or so at LHC. At RHIC (Au-Au collisions at $\sqrt{s} = 200$ GeV) the typical energy density and coupling contstant would be $\epsilon \sim 50$ GeV/fm^3 and $\alpha_S \sim 0.33$ respectively. For this situation one would expect roughly $gA \sim 1$ GeV. So, for these input parameters at RHIC and LHC, the minimum momentum above which one would apply perturbative calculations is about 1 and 2 GeV respectively. It is interesting to note that these values are more or less equal to the minium momentum cut-offs used to compute minijets by using pQCD at RHIC and LHC [29,28]. We have to mention that even if gA is small the contributions of both diagrams have to be taken in order to maintain exact gauge invariance.

It is clear from the above arguments that during the very early stage of heavy-ion collisions our perturbative calculation is applicable for the production of partons having a momentum greater than gA. Begining from such values of the field strength, it is now necessary to investigate how the field decays owing to all effects such as particle production from the field, acceleration of the partons by the field, precession of the color charge in the presence of the field, collision among the partons, and expansion of the system. As time progresses the field strength decreases due to the effects mentioned above. However, as long as gA is greater than Λ_{QCD}, our perturbative method is applicable. The decrease of the product gA with time allows for the description of partons with less minimum momentum than right at the start of the URHIC. The picture breaks down however, when gA reaches Λ_{QCD}. In that case non-perturbative calculations for parton production are needed, which is beyond the scope of this paper. Unfortunately, when trying to perform non-perturbative calculations one always encounters many more obstacles.

How rapidly the boundary of applicability of our picture is reached and how the quark-gluon plasma evolves (up to a time where the condition $gA > \Lambda_{QCD}$ is violated) with our treatment of parton production from the space-time dependent chromofield, must - preferrably - be determined from a self-consisitent transport calculation with all the above effects taken into account. This project will involve a considerable amount of numerical work and will be undertaken in future.

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APPENDIX

Gluon Source Term

We start from definition (37) and introduce the expression (33) and subsequently Eq.(30) to obtain:

$$W_{1A,1A} = \frac{g^2}{32(2\pi)^2} \int \frac{d^3k_1}{k_1^0}\frac{d^3k_2}{k_2^0} d^4K d^4K' \delta^{(4)}(K - k_1 - k_2)\delta^{(4)}(K' - K) A^{a\mu}(K) A^{*a'\mu'}(K')$$
$$f^{abd} f^{a'bd}[-2g_{\mu\rho}K_\nu + g_{\nu\rho}(k_1 - k_2)_\mu + 2g_{\mu\nu}K_\rho][-2g_{\mu'\rho}K'_\nu + g_{\nu\rho}(k'_1 - k'_2)_{\mu'} + 2g_{\mu'\nu}K'_\rho] \quad (87)$$

Now we make use of the Fourier transform of Eq.(6). In the next steps, we first carry out the integration over d^4K' and then over d^4K:

$$W_{1A,1A} = \frac{3g^2}{8(2\pi)^6} \int \frac{d^3k_1}{k_1^0}\frac{d^3k_2}{k_2^0} d^4x d^4x' e^{i(k_1+k_2)\cdot(x-x')} A^{a\mu}(x) A^{a\mu'}(x')$$
$$[2g_{\mu\mu'}(k_1 + k_2)^2 - 2(k_1 + k_2)_\mu (k_1 + k_2)_{\mu'} + (k_1 - k_2)_\mu (k_1 - k_2)_{\mu'}] \quad (88)$$

In the previous step, use has been made of Eq.(38). Now, we extract the expression (40) for the source term. From there, we proceed by replacing the occurrences of k_2 by derivatives with respect to $i(x - x')$:

$$\frac{dW_{1A,1A}}{d^4x d^3k} = \frac{3g^2}{8(2\pi)^6 k^0} \int d^4x' \frac{d^3k_2}{k_2^0} e^{ik\cdot(x-x')} A^{a\mu}(x) A^{a\mu'}(x')[2g_{\mu\mu'}(k + \frac{\partial}{i\partial(x - x')})^2$$
$$-2(k + \frac{\partial}{i\partial(x - x')})_\mu (k + \frac{\partial}{i\partial(x - x')})_{\mu'} + (k - \frac{\partial}{i\partial(x - x')})_\mu (k - \frac{\partial}{i\partial(x - x')})_{\mu'}] e^{ik_2\cdot(x-x')} \quad (89)$$

Now the integration over d^3k_2 can be carried out with the help of Eq. (41) In the end, we explicitly calculate the derivatives and obtain Eq.(42).

Now we carry out the same procedure for the other contributions. First, we calculate the probability of the mixed term. Starting from the definition Eq.(43) we use the expressions (33) and (34) and get by subsequent introduction of Eqs. (30) and (31):

$$W_{1A,2A} = W_{2A,1A} = \frac{-ig^3}{64(2\pi)^4} \int \frac{d^3k_1}{k_1^0}\frac{d^3k_2}{k_2^0} d^4k_3 d^4k_4 d^4K \delta^{(4)}(K - k_3 - k_4)\delta^{(4)}(K - k_1 - k_2)$$
$$A^{a\mu}(k_3) A^{c\lambda}(k_4) A^{*a'\mu'}(K)[-2g_{\mu'\rho}K_\nu + g_{\nu\rho}(k_1 - k_2)_{\mu'} + 2g_{\mu'\nu}K_\rho]$$
$$[f^{a'bd} f^{abx} f^{xcd}(g_{\mu\lambda}g_{\nu\rho} - g_{\mu\rho}g_{\nu\lambda} + g_{\mu\nu}g_{\lambda\rho})$$
$$+f^{a'bd} f^{adx} f^{xbc}(g_{\mu\nu}g_{\lambda\rho} - g_{\mu\lambda}g_{\nu\rho} - g_{\mu\rho}g_{\nu\lambda})$$
$$+f^{a'bd} f^{acx} f^{xbd}(g_{\mu\nu}g_{\lambda\rho} - g_{\mu\rho}g_{\nu\lambda})]. \quad (90)$$

Now, we contract all possible color and Lorentz indices and make use of the Fourier transform (6). From there the source term (45) can be extracted. In the next step, the d^4K and the d^4k_4 integration are carried out, followed by the d^4k_3 integration:

$$\frac{dW_{2A,1A}}{d^4xd^3k} = \frac{-3ig^3}{8(2\pi)^{10}} \int \frac{d^3k_2}{k^0k_2^0} d^4k_3 d^4x_3 d^4x_4 e^{i[k_3\cdot(x_3-x_4)+(k+k_2)\cdot(x_4-x)]}$$
$$A^{a\mu}(x_3)A^{c\lambda}(x_4)A^{a'\mu'}(x)f^{a'ac}(k+k_2)_\lambda g_{\mu\mu'}$$
$$= \frac{-3ig^3}{8(2\pi)^{10}} \int \frac{d^3k_2}{k^0k_2^0} d^4x_3 d^4x_4 (2\pi)^4\delta^{(4)}(x_3-x_4) e^{i(k+k_2)\cdot(x_4-x)}$$
$$A^{a\mu}(x_3)A^{c\lambda}(x_4)A^{a'\mu'}(x)f^{a'ac}(k+k_2)_\lambda g_{\mu\mu'}. \quad (91)$$

The new Dirac δ distribution can be used to perform one of the integrations over d^4x:

$$\frac{dW_{2A,1A}}{d^4xd^3k} = \frac{-3ig^3}{8(2\pi)^6} \int \frac{d^3k_2}{k^0k_2^0} d^4x_3 e^{i(k+k_2)\cdot(x_3-x)} A^{a\mu}(x_3)A^{c\lambda}(x_3)A^{a'\mu'}(x)f^{a'ac}(k+k_2)_\lambda g_{\mu\mu'}. \quad (92)$$

Now, k_2 has got to be replaced by a derivative with respect to $i(x_3-x)$ in order to enable the use of Eq. (41) as a menas to obtain Eq.(46).

The source term of the last contribution remains to be calculated. We receive by introducing Eq. (31) into Eq.(34) and all that into Eq. (48):

$$W_{2A,2A} = \frac{g^4}{128(2\pi)^6} \int \frac{d^3k_1}{k_1^0}\frac{d^3k_2}{k_2^0} d^4k_3 d^4k_4 d^4k_3' d^4k_4'$$
$$\delta^{(4)}(k_1+k_2-k_3-k_4)\delta^{(4)}(k_3+k_4-k_3'-k_4')A^{a\mu}(k_3)A^{c\lambda}(k_4)A^{*a'\mu'}(k_3')A^{*c'\lambda'}(k_4')$$
$$[f^{abx}f^{xcd}(g_{\mu\lambda}g_{\nu\rho}-g_{\mu\rho}g_{\nu\lambda}+g_{\mu\nu}g_{\lambda\rho})+f^{adx}f^{xbc}(g_{\mu\nu}g_{\lambda\rho}-g_{\mu\lambda}g_{\nu\rho}-g_{\mu\rho}g_{\nu\lambda})$$
$$+f^{acx}f^{xbd}(g_{\mu\nu}g_{\lambda\rho}-g_{\mu\rho}g_{\nu\lambda})]\times\ [f^{a'bx'}f^{x'c'd}(g_{\mu'\lambda'}g_{\nu\rho}-g_{\mu'\rho}g_{\nu\lambda'}+g_{\mu'\nu}g_{\lambda'\rho})$$
$$+f^{a'dx'}f^{x'bc'}(g_{\mu'\nu}g_{\lambda'\rho}-g_{\mu'\lambda'}g_{\nu\rho}-g_{\mu'\rho}g_{\nu\lambda'})+f^{a'c'x'}f^{x'bd}(g_{\mu'\nu}g_{\lambda'\rho}-g_{\mu'\rho}g_{\nu\lambda'})] \quad (93)$$

Now, all possible indices have to be contracted:

$$W_{2A,2A} = \frac{g^4}{64(2\pi)^{14}} \int \frac{d^3k_1}{k_1^0}\frac{d^3k_2}{k_2^0} d^4k_3 d^4k_4 d^4k_3' d^4k_4' d^4x_3 d^4x_4 d^4x_3' d^4x_4' e^{i[k_3\cdot x_3+k_4\cdot x_4-k_3'\cdot x_3'-k_4'\cdot x_4']}$$
$$\delta^{(4)}(k_1+k_2-k_3-k_4)\delta^{(4)}(k_3+k_4-k_3'-k_4')A^{a\mu}(x_3)A^{c\lambda}(x_4)A^{a'\mu'}(x_3')A^{c'\lambda'}(x_4')$$
$$[2g_{\mu\lambda}g_{\mu'\lambda'}(f^{abx}f^{xcd}+f^{adx}f^{xcb})(f^{a'bx'}f^{x'c'd}+f^{a'dx'}f^{x'c'b})+24g_{\mu\mu'}g_{\lambda\lambda'}f^{acx}f^{a'c'x}] \quad (94)$$

From this expression we get the source term (49). In the following steps the integrations over all d^4ks and afterwards over d^4x_4 and d^4x_4' are performed:

$$\frac{dW_{2A,2A}}{d^4xd^3k} = \frac{g^4}{32(2\pi)^{14}} \int \frac{d^3k_2}{k^0k_2^0} d^4k_3 d^4k_3' d^4x_4 d^4x_3' d^4x_4' e^{i[k_3\cdot x+(k+k_2-k_3)\cdot x_4-k_3'\cdot x_3'-(k+k_2-k_3')\cdot x_4']}$$
$$A^{a\mu}(x)A^{c\lambda}(x_4)A^{a'\mu'}(x_3')A^{c'\lambda'}(x_4')$$
$$[g_{\mu\lambda}g_{\mu'\lambda'}(f^{abx}f^{xcd}+f^{adx}f^{xcb})(f^{a'bx'}f^{x'c'd}+f^{a'dx'}f^{x'c'b})+12g_{\mu\mu'}g_{\lambda\lambda'}f^{acx}f^{a'c'x}]$$
$$= \frac{g^4}{32(2\pi)^{14}} \int \frac{d^3k_2}{k^0k_2^0} d^4x_4 d^4x_3' d^4x_4' e^{i(k+k_2)\cdot(x_4-x_4')}(2\pi)^4\delta^{(4)}(x-x_4)(2\pi)^4\delta^{(4)}(x_3'-x_4')$$
$$A^{a\mu}(x)A^{c\lambda}(x_4)A^{a'\mu'}(x_3')A^{c'\lambda'}(x_4')$$
$$[g_{\mu\lambda}g_{\mu'\lambda'}(f^{abx}f^{xcd}+f^{adx}f^{xcb})(f^{a'bx'}f^{x'c'd}+f^{a'dx'}f^{x'c'b})+12g_{\mu\mu'}g_{\lambda\lambda'}f^{acx}f^{a'c'x}]$$
$$= \frac{g^4}{32(2\pi)^6} \int \frac{d^3k_2}{k^0k_2^0} d^4x_3' e^{i(k+k_2)\cdot(x-x_3')} A^{a\mu}(x)A^{c\lambda}(x)A^{a'\mu'}(x_3')A^{c'\lambda'}(x_3')$$
$$[g_{\mu\lambda}g_{\mu'\lambda'}(f^{abx}f^{xcd}+f^{adx}f^{xcb})(f^{a'bx'}f^{x'c'd}+f^{a'dx'}f^{x'c'b})+12g_{\mu\mu'}g_{\lambda\lambda'}f^{acx}f^{a'c'x}] \quad (95)$$

For the integration over d^3k_2 we again make use of Eq. (41) and obtain the final expression Eq.(50).

Gluon source term for the time-dependent case

In the case were the A fields are only time dependent, we receive from Eq. (40) by integrating over the spatial coordinates:

$$\begin{aligned}\frac{dW_{1A,1A}}{d^4xd^3k} = \frac{3g^2}{8(2\pi)^3}\int \frac{d^3k_2}{k^0k_2^0}dt'\delta^{(3)}(\vec{k}+\vec{k}_2)e^{i[(k+k_2)\cdot x-(k^0+k_2^0)t']}A^{a\mu}(t)A^{a\mu'}(t')\\ [2g_{\mu\mu'}(k+k_2)^2-2(k+k_2)_\mu(k+k_2)_{\mu'}+(k-k_2)_\mu(k-k_2)_{\mu'}]\end{aligned} \tag{96}$$

By eliminating all Dirac δ distributions one obtains:

$$\begin{aligned}\frac{dW_{1A,1A}}{d^4xd^3k} = \frac{3g^2}{8(2\pi)^3 4(k^0)^2}\int dt' e^{2ik^0(t-t')}\\ 4[-2(\vec{A}^a(t)\cdot\vec{A}^a(t'))(k^0)^2+(\vec{A}^a(t)\cdot\vec{k})(\vec{A}^a(t')\cdot\vec{k})],\end{aligned} \tag{97}$$

where the result (71) can be obtained incorporating Fourier transforms.

As before, we integrate in Eq. (45) over all spatial coordinates and subsequently integrate over all d^4ks followed by dt_4:

$$\begin{aligned}\frac{dW_{2A,1A}}{d^4xd^3k} &= \frac{-3ig^3}{8(2\pi)^4}\int \frac{d^3k_2}{k^0k_2^0}d^4k_3d^4k_4d^4Kd^4t_3d^4t_4\delta^{(4)}(K-k_3-k_4)\delta^{(4)}(K-k-k_2)\\ &\quad \delta^{(3)}(\vec{k}_3)\delta^{(3)}(\vec{k}_4)e^{i[k_3^0t_3+k_4^0t_4-K\cdot x]}A^{a\mu}(t_3)A^{c\lambda}(t_4)A^{a'\mu'}(t)f^{a'ac}K_\lambda g_{\mu\mu'}\\ &= \frac{-3ig^3}{8(2\pi)^4}\int \frac{d^3k_2}{k^0k_2^0}d^4k_3d^4t_3d^4t_4\delta^{(3)}(\vec{k}_3)\delta^{(3)}(\vec{k}+\vec{k}_2)e^{i[k_3^0t_3+(k^0+k_2^0-k_3^0)t_4-(k+k_2)\cdot x]}\\ &\quad A^{a\mu}(t_3)A^{c\lambda}(t_4)A^{a'\mu'}(t)f^{a'ac}(k+k_2)_\lambda g_{\mu\mu'}\\ &= \frac{-3ig^3}{8(2\pi)^4}\int \frac{d^3k_2}{k^0k_2^0}d^4t_3d^4t_4\delta^{(3)}(\vec{k}+\vec{k}_2)(2\pi)\delta(t_3-t_4)e^{i[(k^0+k_2^0)t_4-(k+k_2)\cdot x]}\\ &\quad A^{a\mu}(t_3)A^{c\lambda}(t_4)A^{a'\mu'}(t)f^{a'ac}(k+k_2)_\lambda g_{\mu\mu'}\\ &= \frac{-3ig^3}{8(2\pi)^3(k^0)^2}\int dt_3e^{2ik^0(t_3-t)}A^{a\mu}(t_3)A^{c0}(t_3)A^{a'\mu'}(t)f^{a'ac}2k^0g_{\mu\mu'}\\ &= \frac{-3ig^3}{4(2\pi)^3k^0}\int dt_3e^{2ik^0(t_3-t)}A^{a\mu}(t_3)A^{c0}(t_3)A^{a'\mu'}(t)f^{a'ac}g_{\mu\mu'}.\end{aligned} \tag{98}$$

Starting from Eq. (49), we carry out similar steps for the last gluon term:

$$\begin{aligned}\frac{dW_{2A,2A}}{d^4xd^3k} = \frac{g^4}{32(2\pi)^5}\int \frac{d^3k_2}{k^0k_2^0}d^4k_3d^4k_4d^4k_3'd^4k_4'dt_4dt_3'dt_4'e^{i[k_3\cdot x+k_4^0t_4-k_3'^0t_3'-k_4'^0t_4']}\\ \delta^{(3)}(\vec{k}_4)\delta^{(3)}(\vec{k}_3')\delta^{(3)}(\vec{k}_4')\\ \delta^{(4)}(k+k_2-k_3-k_4)\delta^{(4)}(k_3+k_4-k_3'-k_4')A^{a\mu}(t)A^{c\lambda}(t_4)A^{a'\mu'}(t_3')A^{c'\lambda'}(t_4')\\ [g_{\mu\lambda}g_{\mu'\lambda'}(f^{abx}f^{xcd}+f^{adx}f^{xcb})(f^{a'bx'}f^{x'c'd}+f^{a'dx'}f^{x'c'b})+12g_{\mu\mu'}g_{\lambda\lambda'}f^{acx}f^{a'c'x}]\end{aligned}$$

$$
\begin{aligned}
&= \frac{g^4}{32(2\pi)^5}\int \frac{d^3k_2}{k^0k_2^0}d^4k_3d^4k_3'dt_4dt_3'dt_4'e^{i[k_3\cdot x+(k^0+k_2^0-k_3^0)t_4-k_3't_3'-(k^0+k_2^0-k_3'^0)t_4']}\\
&\delta^{(3)}(\vec{k}+\vec{k}_2-\vec{k}_3)\delta^{(3)}(\vec{k}_3')\delta^{(3)}(\vec{k}+\vec{k}_2-\vec{k}_3')A^{a\mu}(t)A^{c\lambda}(t_4)A^{a'\mu'}(t_3')A^{c'\lambda'}(t_4')\\
&[g_{\mu\lambda}g_{\mu'\lambda'}(f^{abx}f^{xcd}+f^{adx}f^{xcb})(f^{a'bx'}f^{x'c'd}+f^{a'dx'}f^{x'c'b})+12g_{\mu\mu'}g_{\lambda\lambda'}f^{acx}f^{a'c'x}]\\
&= \frac{g^4}{32(2\pi)^5}\int \frac{d^3k_2}{k^0k_2^0}dt_4dt_3'dt_4'e^{i(k^0+k_2^0)(t_4-t_4')}\\
&\delta^{(3)}(\vec{k}+\vec{k}_2)(2\pi)\delta(t-t_4)(2\pi)\delta(t_3'-t_4')A^{a\mu}(t)A^{c\lambda}(t_4)A^{a'\mu'}(t_3')A^{c'\lambda'}(t_4')\\
&[g_{\mu\lambda}g_{\mu'\lambda'}(f^{abx}f^{xcd}+f^{adx}f^{xcb})(f^{a'bx'}f^{x'c'd}+f^{a'dx'}f^{x'c'b})+12g_{\mu\mu'}g_{\lambda\lambda'}f^{acx}f^{a'c'x}]\\
&= \frac{g^4}{32(2\pi)^3(k^0)^2}\int dt_3'e^{2ik^0(t-t_3')}A^{a\mu}(t)A^{c\lambda}(t_3)A^{a'\mu'}(t_3')A^{c'\lambda'}(t_3')\\
&[g_{\mu\lambda}g_{\mu'\lambda'}(f^{abx}f^{xcd}+f^{adx}f^{xcb})(f^{a'bx'}f^{x'c'd}+f^{a'dx'}f^{x'c'b})+12g_{\mu\mu'}g_{\lambda\lambda'}f^{acx}f^{a'c'x}].
\end{aligned}
\tag{99}
$$

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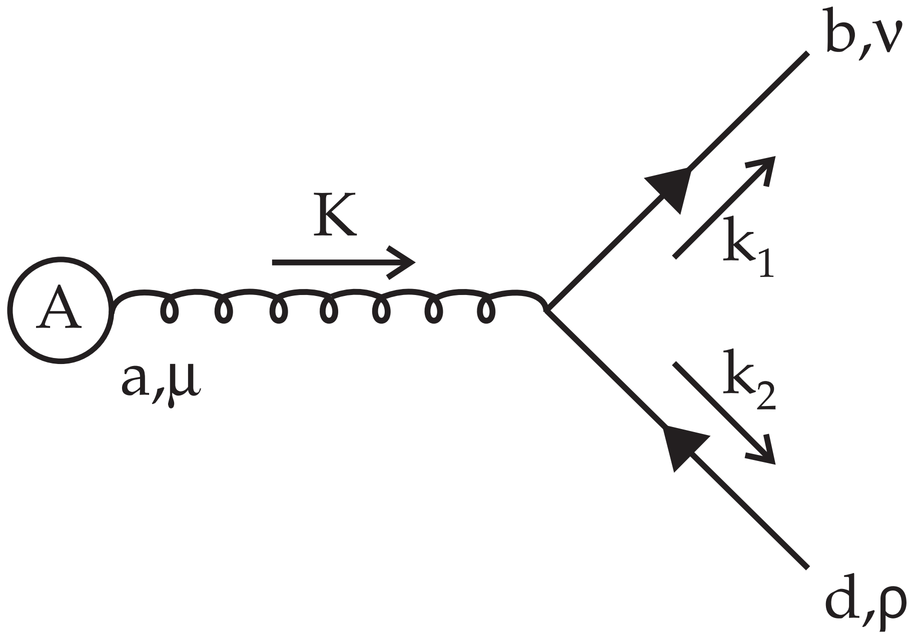

Fig. 1 Feynman diagram for the production of an quark-antiquark pair by coupling to the field A once.

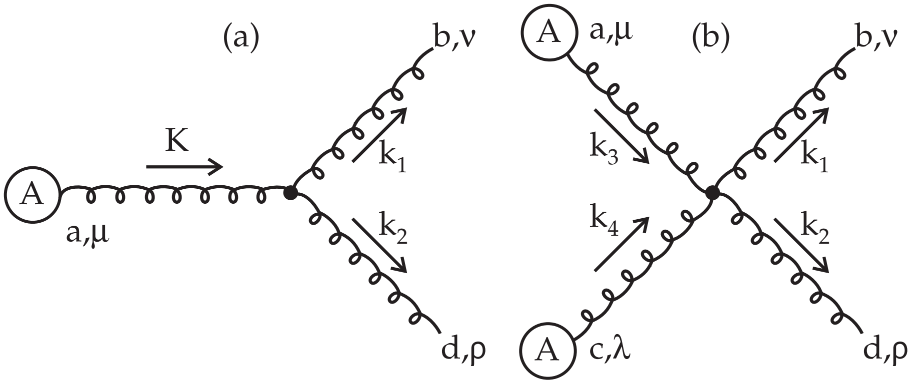

Fig. 2 Feynman diagrams for the production of two gluons by coupling to the field A once (a) or twice (b).

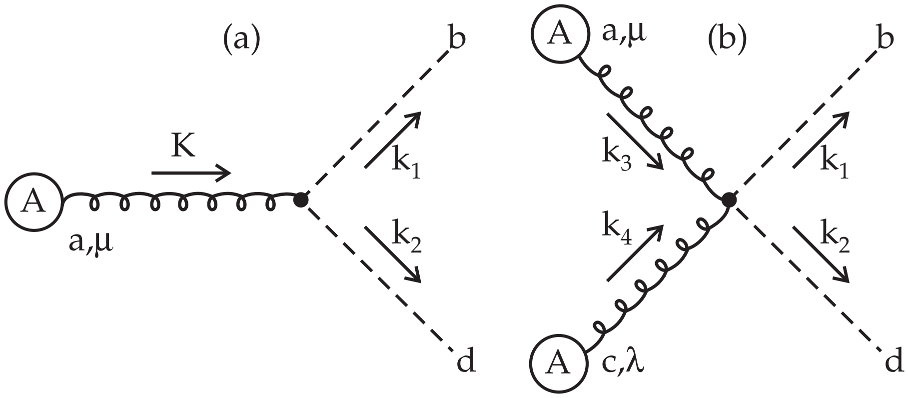

Fig. 3 Feynman diagrams for the ghosts, corresponding to the gluon vertices in Fig.(2).

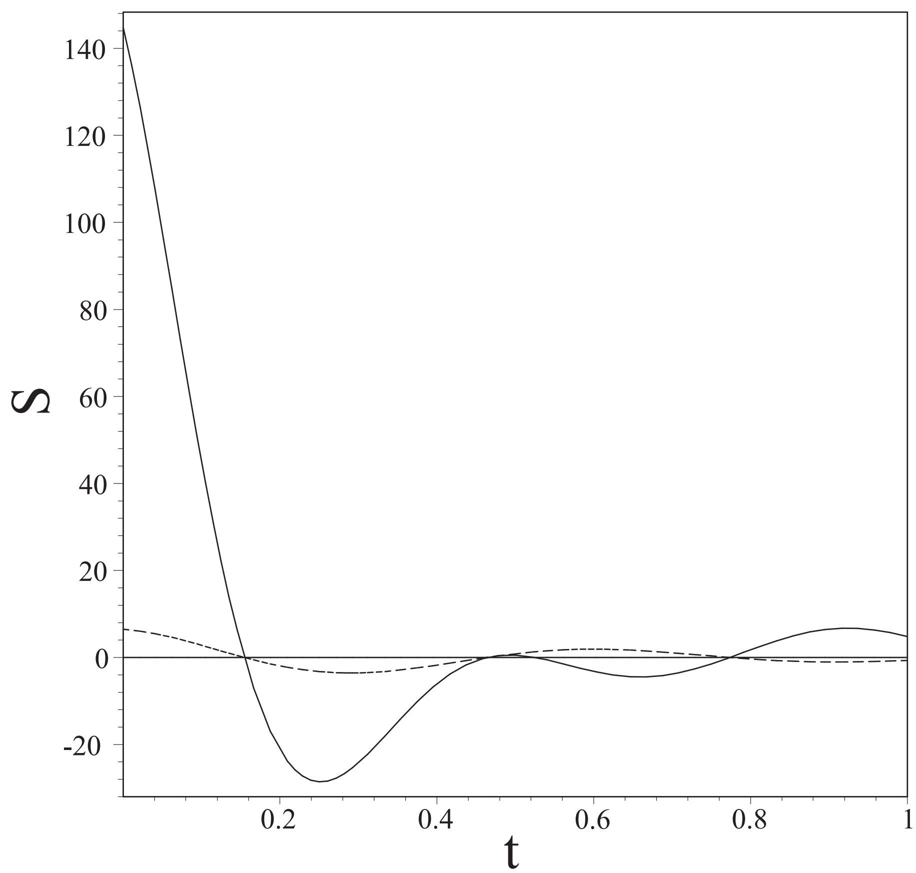

Fig. 4 Source terms S in MeV for the production of gluon pairs (solid line) and quark-antiquark pairs (dashed line) versus time t in fm.

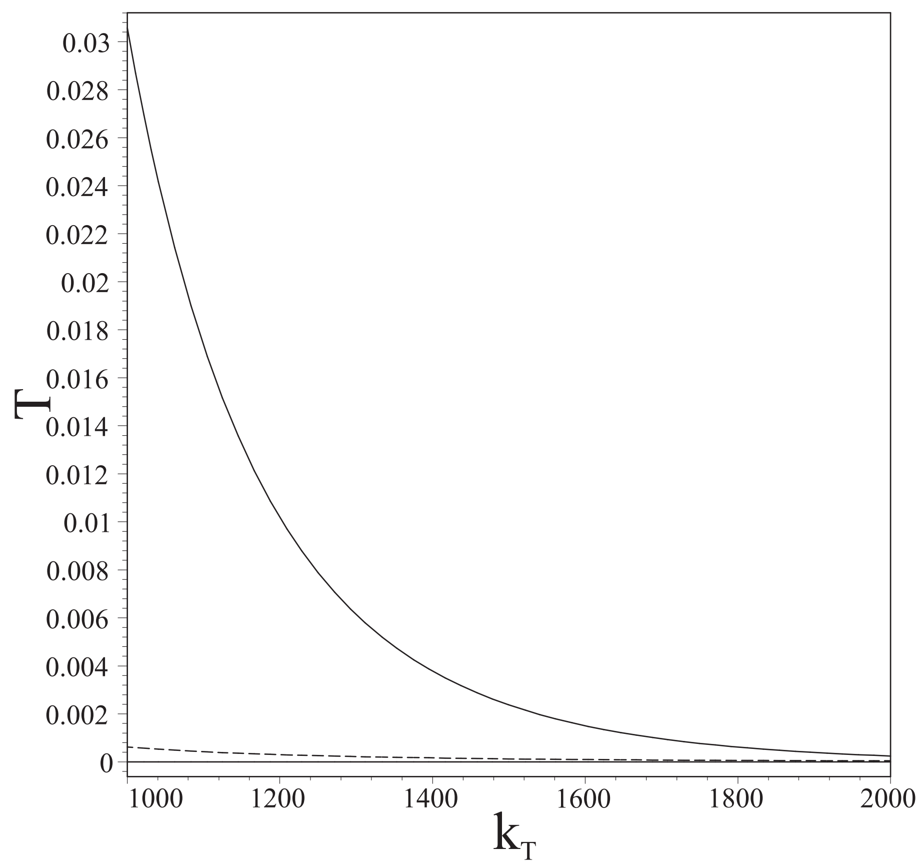

Fig. 5 The dimensionless time-integrated source terms T for the production of gluon pairs (solid line) and quark-antiquark pairs (dashed line) versus transverse momentum k_T in MeV.

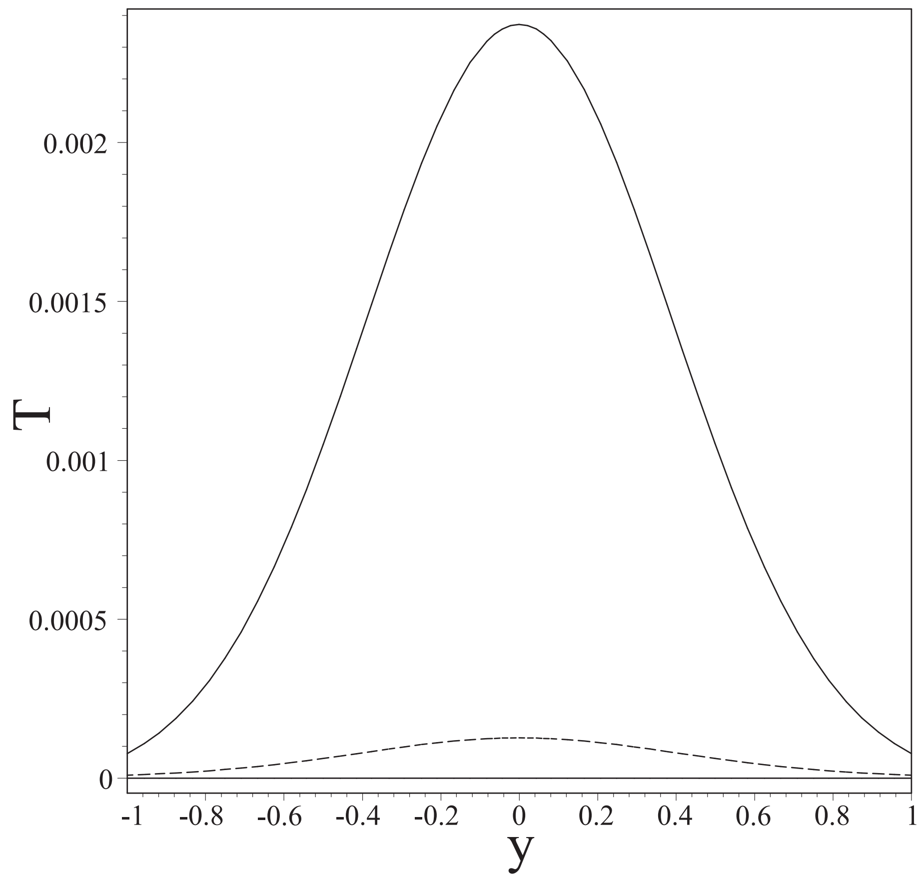

Fig. 6 The dimensionless time-integrated source terms T for the production of gluon pairs (solid line) and quark-antiquark pairs (dashed line) versus rapidity y.

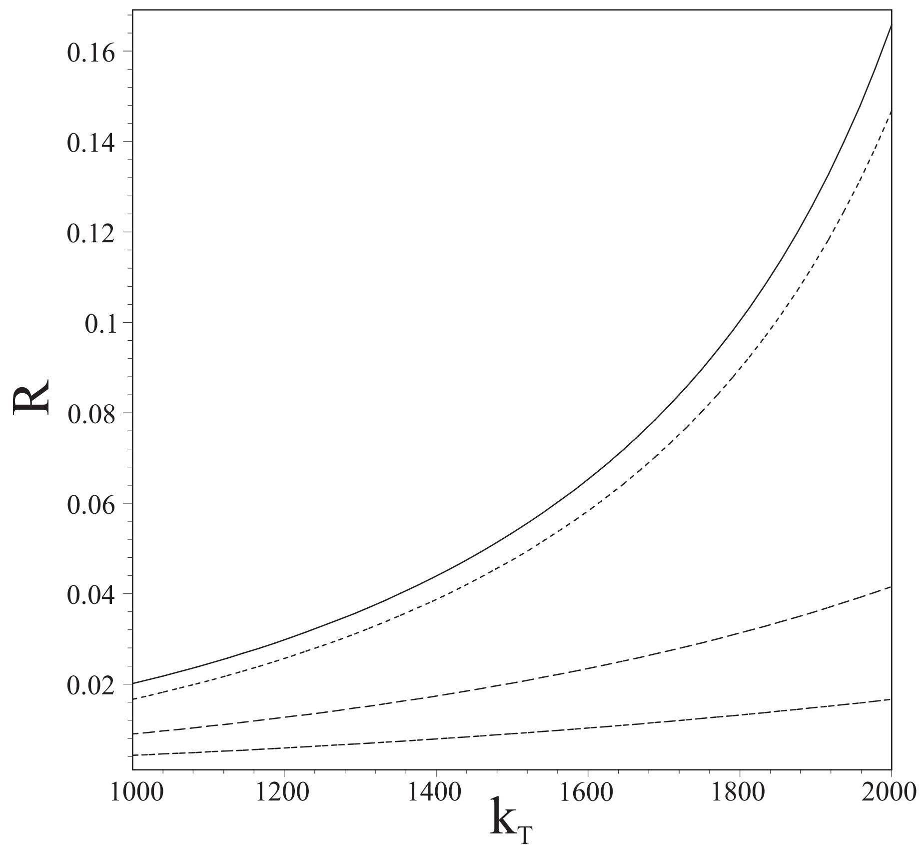

Fig. 7 Ratio R of the time-integrated source terms for the production of gluon pairs and quark-antiquark pairs versus transverse momentum k_T in MeV. From top to bottom we have first the graph for the standard parameters: coupling constant $\alpha_S = 0.15$, initial field strength $A_{in} = 1.5GeV$, and decay time $t_0 = 0.5fm$. In the subsequent graphs we increased the coupling constant to $\alpha_S = 0.3$, the initial field strength to $A_{in} = 3GeV$, and the decay time to $t_0 = 1fm$, respectively.